**FINANCIAL MATHEMATICS PROJECT**

A Dissertation

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Submitted by

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**CERTIFICATE**

This is to certify that Mr. Digambar Prasad Singh Roll No. 1809014 Enrolment No. 181020 is registered for the Integrated Master’s Program in the Department of Mathematics of the National Institute of Technology Patna

I hereby recommend that the Dissertation entitled “Financial Mathematics” be accepted as the partial fulfillment of the requirements of evaluation and award of the Integrated Master of Science Degree by the National Institute of Technology Patna

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(Signature with Name,

Designation and Seal)

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**Abstract**

This project describes the underlying application of Financial Mathematics, the present value, future, and the present value of the annuity, Sinking fund, perpetuity, Bonds, and financial derivatives. This project helps in the understanding of derivatives, like-future, forwards, swaps, and options. Option pricing uses different methods and also determines a radial basis function-based implicit-explicit method for option pricing under jump-diffusion models. This project helps in understanding the financial things and can apply in real life and can invest in the market.

INTRODUCTION:

Financial Mathematics is the application of mathematical methods to financial problems. (Equivalent names sometimes used are quantitative finance, financial engineering, mathematical finance, and computational finance.) It draws on tools from probability, statistics, stochastic processes, and economic theory.

**Prominence of Financial Mathematics**

substantially in the past, and such a trend is expected to continue. Various types of organizations and the use of mathematics and statistics within the field of finance has been increasing financial service providers utilize financial mathematics as part of their core operations, such as: Investment banks

* Retail and commercial banks
* Hedge funds
* Investment management companies
* Corporate treasuries
* Regulatory bodies

In addition, financial mathematics is applied considerably to solve problems, such as:

* Derivative security pricing and valuation
* Portfolio creation and structuring
* Quantitative investing strategies
* Risk management

BASIC THINGS THAT WE USED IN FINANCE

**Present value**: If the money is worth i per periods, then present value P of amount S due n periods hence is given by P=S(1+i)-n

**Future value of an annuity**: The amount of future value S of an ordinary annuity of Rs R per period for n periods at the rate i per period is given by:

S=R

**Present value of an annuity:**

The present value P of an ordinary annuity of Rs R per payment periods for n periods at the rate of i per period is given by

P=R

***Sinking fund****: Sinking funds are funds that are periodically accumulated by the company as reserve. Later the reserve fund is used for a specific purpose—repayment of debts or repurchase of bonds on maturity. As a result, companies are not burdened with paying a huge sum at once.*

*the rate i per period is given by*

*R=*

*a new equipment in 5 years. How much should be deposited compounded quarterly to provide for the purchase?*

*Given, S=50,000, n=5\*4, and i = =0.03*

*R**=*

*=*

*=*

*Hence the required amount=1860.81*

***Perpetuity****: A perpetuity is a special form of an ordinary annuity in which period payment continue forever.*

*There are two types of perpetuity:*

*1.****Perpetuity payable at the end of each payment period.***

*Consider a perpetuity of R payable at the end of each period, the first payment period due one period hence. Let the money be I per period .The present value of this perpetuity is defined to be that sum of money which ,invested now at the rate I per period ,will yield R at the end of each period forever.*

*The present value of first payment=**R(1+i)-1*

*The present value of second payment=R(1+i)-2*

*The present value of third payment=R(1+i)-3 and so on.*

*So, the present value P of the perpetuity is given by*

*P= R(1+i)-1 +R(1+i)-2 + R(1+i)-3……+…...+……*

*This is forming infinite terms of G.P with*

*First term= R(1+i)-1*

*Common ratio=(1+i)-1*

*P =*

***2. Perpetuity payable at the beginning of each payment period****.*

*The present value of first payment=R*

*The present value of second payment=R(1+i)-1*

*The present value of third payment=R(1+i)-2 and so on*

*Similarly, as above, we get*

*P=R+*

*EX-Find the present value of a perpetuity of Rs 3,120 payable at the beginning of each year, if money is worth 6% effective?*

*Given, R =3120 and i = 0.06*

*Let P be the present value of this annuity.*

*P=R+*

*=3,120 + =55,120*

*Hence the present value is Rs 55,120.*

***Bond****: A bond is debt instrument that provides a periodic streams of interest payments to investor (or buyers) while repaying the principal sum on specific maturity date.*

***Maturity****: A bonds maturity is the time period until the principal is schedule to be repaid.*

***Face value****: The face value (also known as par value) of a bond is the price at which the bond is sold to investors (or buyers) at the time of issue.*

***Redemption price****: The redemption price of a bond is the amount the bond issuer pays at maturity.*

***Dividend rate****: The rate at which a bond yields interest is called the dividend rate or the nominal interest rate.*

***Coupon rate****: A bond’s coupon rate denotes the annual interest rate paid by the bond issuer to the bond holder.*

***Fare Price****: The fair price of a bond is the open market value that is acceptable to both the buyer and the seller*

*Let there be a bond with the*

*Face Value=F*

*Redemption Price or Maturity value=C*

*Yield rate =i*

*Number of periodic payments =n*

*Periodic interest payment or coupon payment=R*

*Value of bond =V*

Bond Value =Present value of first periodic payment +………+………+present value of nth periodic payment +redemption price.

Bond Value= **+ +……+** +

=

V =R-n

*EX-A company issued a bond having a face value of Rs 100,000, carrying an annual dividend rate 7% and maturing in 15 years. If prevailing market rate of interest is 9%, and the bond is redeemed at par, find the bond value. Given( )*

*F=100,000*

*n=15, i =0.09*

*R =Annual dividend=7% of face value=100,000\*0.07=7,000*

*The bond is redeemed at par*

*C = F = 100,000*

*V=*

*=*

*= (56424.819 +27453.804)*

*=83878.62*

*Hence the bond value is Rs 83,878.62*

***EMI CALCULATION***

**EMI:** An equated monthly installment (EMI) is a fixed payment made by a borrower to a lander on a specific date of each month.

EMI depends upon the following factors:

(i)The amount of loan (ii)The loan tenure (iii)The interest rate.

* EMI can be calculated by two methods:

(i)Flat Rate Method (FRM) (ii)Reducing Balance Method (RBM)

**(i)Flat Rate Method**: In flat rate method the principal amount remains same throughout the tenure and the interest is charged on it at a constant rate throughout the tenure. Suppose an amount of P is borrowed at flat rate of r per rupee per month for a period of n months. Then,

Interest=Pin

EMI=

*=P=P*

*EX- Mira takes a loan of rupees 300,000 at an interest of 10%* compounded annually for a period of 3 years. Find her EMI by using flat rate method.

Sol: We have P=300,000

i= Rate of interest per rupee per month= =

n= number of installments=12\*3=36.

EMI= *P=*300,000=10,833.333

Hence Mira’s EMI is of 10,833.333

**(ii)Reducing Balance Method (RBM):** In reducing balance method principal paid back gets deducted from the outstanding load amount and the interest for the subsequent year is charged on the remaining deducted balance and not on the entire loan amount unlike the flat rate method.

Let the amount P borrowed on which interest is payable at the rate of r per rupee per installment period. This amount is to be paid along with the interest in n equal installments of Rs E each. Each installment comprises some part of the principal P that is to be paid back; the rest is the interest on the amount that was outstanding for that installment period. Let Pn denote the principal amount left (outstanding amount) at the end of nth month.

Initially i.e., at the beginning of the tenure: We have

i=0 and P0=P=Principal amount

First month: We have,

Principal=P, Installment paid=E, Interest accrued=Pi

Outstanding amount= (P+ Pi)-E

P1=P(1+i)-E

Second month:

Principal=Outstanding amount at the end of second month=P1

Installment paid= E, Interest accrued=P1i

Outstanding amount=(P1+P1i)-E

=P1(1+i)-E

=

P2 = P(1+i)2 -

Continuing in this manner, we find that the outstanding at the end of nth month is given by

Pn=P(1+i)n-

Outstanding at the end of nth month is zero i.e., Pn=0

P(1+i)n =E

P (1+i) n =E

E= E=

This gives the formula for calculating EMI, where

E= Equated monthly installment

P= Principal or loan amount

i= Interest rate per rupees per month (the annual interest rate percent divided by 1200)

n= number of monthly installments

EX-Aman borrowed a home loan amount of Rs 5,000,000 from a bank at an interest rate of 12% per annum for 30 years. Find the monthly installment amount Aman has to pay to the bank. (Given, ()).

Sol: We have,

P=5,000,000, i==0.01 and n =12\*30=360

Let E be the monthly installment Aman has to pay to the bank. Then,

E= = =51,430.63

Hence Aman’s EMI is Rs 51430.63

*FINANCIAL DERIVATIVES*

***Derivative****: A derivative is financial contract that derives its value from an underlying asset.*

* *The buyer agrees to purchase the asset on a specific date at a specific price.*
* *Derivatives can be used either for risk management or for speculation (batting)*

*Types of derivatives*

* *Futures*
* *Forwards*
* *Swaps*
* *Options*

***Futures:*** *A future contract is an agreement between two parties to buy or sell an asset at a certain time in the future at a certain price.*

*Ex. 28April 2022 30April 2023*

*(Today price Rs 40) ( want to buy in this day)*

*I.e., After one year we want to*

*buy in same price of Rs 40.*

* *Futures contract to hedge against risk or speculate on the price movement of the underlying asset.*
* *Futures are standardized contracts traded on a centralized exchange.*

***Forward****: A forward contract is a non-standardized contract between two parties to buy or to sell an asset at specified future time at a price argued upon today.*

* *The key difference being that unlike future forward contracts (or forward) are not traded on exchange, rather only over the counter.*
* *Forwards are used to hedge risk in commodities interest rates, exchange rates or equation.*

***Swaps****: A swap is most often a contract between two parties agreeing to trade loan terms.*

*Let a Home Loan*

*M1 M2*

*SBI -variable interest rate Axis-fixed interest rate*

* *One might use an interest rate swap to switch from a variable interest rate loan to fixed interest rate loan, or vice versa.*
* *The loans will remain in the original holders names the contracts mandates that each party will make payments towards the others loan at mutually agreed upon rate.*

***Options****: A option is similar to a future contract in that it is an agreement between two at parties granting on the opportunity to buy or sell a security from or to the other party predetermined future date.*

* *The key difference between options and futures is that option, simply gives the buyer the option to either buy or sell the asset at a certain price and date.*

*Types of options:*

1. *Call options: “Call” gives the buyer the right but not the obligation to buy a given quantity of the underlying asset, at a given price on or before a given future date.*
2. *Put options:” Put” gives the buyer the right, but not the obligation to sell a given quantity of underlying asset at a given price on or before a given future date. All the options contracts are settled in cash.*

* *Options are classified based on type of exercise.*

1. *European option*
2. *American option*

***1.European option****: European option are option that can be exercised only on the expiration date.*

* *All index option traded at NSE is European options.*

***2.American options****: American options are option contract that can be exercised at any time up to the expiration date.*

* *Options on individual securities available at national stock exchange are American type of options.*

*Option Pricing or Valuation of Options*

*Black Scholes Mode l(BSM)*

*As per BSM, value of option is calculated as under*

*C0 =S0\*n(d1) -(d2)*

*S0 = Current market price*

*n(d1) =Delta of option or probability of stock price more*

*than exercise price*

*n(d2) = probability of exercise of option*

*E =Exercise price*

*r =rate of risk-free interest*

*t =time*

*EX- S0 =Rs 550, E =490, t =6 months (0.5 years), r =10%p.a.*

*n(d1) = 0.859, n(d2) = 0.7245.*

*Value of call option:*

*As per BSM:*

*C0 =S0\*n(d1) - \*n(d2)*

*= 550\*0.8592 - \*0.7245*

*= 472.56 -339.07 =133.49*

* *d1 and d2 is calculated as under:*

***d1 =***

***d2 =d1 -***

***d1= Z=*** *This is takenfrom normal probability distribution.*

*EX-Current market price =Rs 165*

*Exercise price =Rs 150*

*Risk free rate of interest =6 % p.a.*

*Calculate*

*1.value of call option*

*2.value of put option*

*Given, S0=165, E =150, r= 6%, t =2 years,*

*d1=*

*d1 = 1.121*

*d2=d1 -*

*=1.121 -0.2121*

*d2 =0.9089*

*From the graph of normal distribution*

*n(d1) =0.8687*

*n(d2) =0.8182*

*Value of call option:*

*C0 =S0\*n(d1) - \*n(d2)*

*=165\*0.8687 -*

*=143.33 -*

*C0 = 34.47*

*Value of Put option*

*P0 =\*n(-d2) -S0\*n(-d1)*

*=\*0.1818 – 165\*0.1313 P0 =2.52*

A radial basis function based implicit–explicit method for option pricing under jump-diffusion models

**INTRODUCTION**

There is evidence to suggest that the Black Scholes model for stock price behavior does not always model real stock price behavior. Jump can appear at a random time and these jumps cannot be captured by the log normal distribution characteristic of the stock price in the Black Scholes model. To overcome the above shortcoming, several models have been proposed. Among these, the jump diffusion models introduced by Merton and Kou are of the most widely used models. Merton proposed a log-normally distributed process for the jump-amplitudes, whereas Kou suggested log-double-exponentially distributed process.

The valuation of option under jump diffusion process requires the solution of a partial integro-differential equation containing a non-local integral term. There are several numerical methods available in the literature to approximate the above equation. In, Almendral and Osterlee presented an implicit second order accurate time discretization with finite difference and finite element spatial discretization on uniform grid. Andersen et al.  proposed an unconditionally stable alternating direction implicit method for its solution. Song Wang et al.  developed a fitted finite volume method for jump diffusion process. Their method is based on fitted finite volume method spatial discretization and Crank Nicolson scheme for temporal discretization. More recently, Patidar et al.  developed an efficient method for pricing Merton jump diffusion option, combining the spectral domain decomposition method and the Laplace transform method. The scheme proposed by d Halluin et al. required to use an iterative procedure to solve discrete equations. The main difficulty with implicit scheme is due to containing non-local integral term in governing equation, which leads to a dense discretization matrix whereas fully explicit scheme-imposed stability restriction on it. An approach based on implicit–explicit schemes in which integral term is treated explicitly has been proposed by YongHoon Kwon et al. and Briani et al. More recently Tangman et al. introduced a new scheme called exponential time integration (ETI) scheme to solve the PIDE. In ETI scheme, the time direction of PIDE is directly tackled by a ‘one step’ formula, which means temporal discretization is not required. Tangman et al. used the central difference approach with ETI to provide very efficient and second order accurate result.

Recently, a new method based on radial basis function (RBF) for approximation of spatial derivative in option pricing equation is undergoing active research. Application of RBF in one dimension European and American options is given by Hon et al. Fasshauer et al. solved American option pricing model using penalty method.

Golbabai et al., developed an algorithm based on global collocation for jump diffusion process. Bhuruth et al. proposed a radial basis function based differential quadrature rule for spatial discretization with exponential time integration to solve jump diffusion model. In more recent work, Chan et al. used new RBF called cubic spline as basis function to solve PIDE, and show that their scheme is second order accurate.

It was recognized that standard approach to solving the radial basis function collocation problem has been ill conditioned due to use of collocation in global sense. Recently many strategies have been developed in the literature to avoid these problems, such as local RBF approach by Lee et al., radial point interpolation method proposed by Liu et al., Shu et al.  proposed a local radial basis function-based differential quadrature method and used it to solve two-dimensional incompressible Navier–Stokes equations. Tolstykh, Tolstykh and Shirobokov, Wright et al. proposed radial basis function finite difference method, the idea is to use radial basis functions with a local collocation as in finite difference mode thereby reducing number of nodes and hence producing a sparse matrix. This technique is further extended by Sanyasi Raju et al. for convection diffusion type equations. However, these methods have not been extended to solve partial integro differential equation yet. In the present work, we have extended the localization concept proposed by Wright and Fornberg, to solve jump diffusion models. The governing equations are discretized by a three-level implicit and explicit time scheme followed by RBF based finite difference method.

The paper is organized as follows. In section [2](https://www.sciencedirect.com/science/article/pii/S0168927416301568?casa_token=lj84pfRyV4kAAAAA:NxhEeS8k4qUnGlFpiMBnp6pRvbB08YmvulU4oza1Bcoe_n6TYDczNI2Cd5GXQQVkl92d3O4Crno" \l "se0020), mathematical models for pricing option with jump diffusion process are given in terms of partial integro-differential equations and provide a brief review of both the Merton and Kou jump diffusion models. Section [3](https://www.sciencedirect.com/science/article/pii/S0168927416301568?casa_token=lj84pfRyV4kAAAAA:NxhEeS8k4qUnGlFpiMBnp6pRvbB08YmvulU4oza1Bcoe_n6TYDczNI2Cd5GXQQVkl92d3O4Crno" \l "se0030) deals with the construction of three time level implicit explicit scheme to discretize the jump diffusion model. The time semi discrete equation is coupled with radial basis function based finite difference method for spatial discretization. Section [4](https://www.sciencedirect.com/science/article/pii/S0168927416301568?casa_token=lj84pfRyV4kAAAAA:NxhEeS8k4qUnGlFpiMBnp6pRvbB08YmvulU4oza1Bcoe_n6TYDczNI2Cd5GXQQVkl92d3O4Crno" \l "se0060) provides extension of proposed method for pricing American option by utilizing concept of operator splitting method. In section [5](https://www.sciencedirect.com/science/article/pii/S0168927416301568?casa_token=lj84pfRyV4kAAAAA:NxhEeS8k4qUnGlFpiMBnp6pRvbB08YmvulU4oza1Bcoe_n6TYDczNI2Cd5GXQQVkl92d3O4Crno" \l "se0070), we give some numerical results for Merton and Kou model and a comparison of the accuracy of our solution with finite difference and finite element method for both American and European options. Finally, the paper ends with some conclusive remarks in section

***MATHEMATICAL MODELS:***

In this section, we give brief discussion about the mathematical model for option with jump diffusion process. Consider an asset with the asset price *S*, then the movement of stock price is modeled by the following stochastic differential equation.

= (ν−κ λ) d τ+ σ d Z+(η−1) d q (1)

where, *ν* is drift rate, *τ* as the time to maturity, *σ* represents the constant volatility,

*d Z* is an increment of standard Gauss–Wiener process. The term *λ* is the intensity of the independent Poisson process *d q* with

d q= {0 with probability 1−λdτ,

{1 with probability λ d τ.

The expected relative jump size E(η−1) is denoted by *κ*, where E [⋅] is the expectation operator and η−1 are an impulse function producing jump from *S*-to-S *η*.

Let V (S, τ) represent the value of a contingent claim that depends on the underlying asset price *S* with current time *τ*. Then V (S, τ) satisfy following backward partial integro differential equation.

++(r−λ κ) S − (r+ λ) V+ λ  = 0, (2)

for (S, τ) ∈ (0, ∞) × (0, T], where, *r* is risk free interest rate and g(η) is probability density function of the jump with amplitude *η* with properties that ∀*η*, g(η)≥0 and =1.

The value of *V* at the expiry date is given by,

V (S, T) =G(S), S ∈ (0, ∞), (3)

where G(S) is the pay-off function for the option contract. Under Merton's model g(η) is given by the log-normal density.

g(η): = (4)

In this case κ: =E (η −1) =exp (μ J+)−1, where μ J and σJ2 are the mean and the variance of jump in return. Under a modified version of Kou's jump-diffusion model g(η) is the following log-double-exponential density

g(η): =(pη1−H(ln(η)) +qη2H(−ln(η))),

where η1>1, η2>0, p>0, q=1−p, and H (⋅) is the Heaviside function. We can show that

 κ:+ − 1. (5)

By using change of variable x= ln (), y=ln(η), where *K* is the strike price and letting t=T−τ, computation of the option value requires solving the PIDE

++ (r− - λ κ) − (r+ λ) u+ λ (6)

where u (x, t): =V(K,T−t) and f(y)=g (), under the above transformation the function f(y) can be written as;

f(y): = Merton’s model (7)

= {pη1H(y) +qη2H (−y) Kou’s model.

The initial condition and asymptotic behavior of the price of a European put option is described by

U (x, 0) =max (K−K ,0)

u (x, t) = {K− K x→−∞, (8)

{0 x→∞. (9)

Other types of initial and boundary conditions can be suitably defined for different types options.

In contrast, the American option can be exercised at any time up to the maturity date and can be formulated as the linear complementarity problem (LCP) of the form

{ – Lu ≥0,

{u (x, t) −G(x)≥0, (10)

−Lu) (u (x, t) −G(x)) =0,

for (x, t) ∈ (0, ∞) ×[0, T) together with the boundary conditions.

u (x, t) ={K−K x→−∞,

{0 x→∞, (11)

where L is the partial integro differential operator on the right side of 6.